Vertical Asymptotes

Remember from Precalculus 1 that
the graph of a function $y = \dfrac{N(x)}{D(x)}$ will usually have a vertical asymptote at $x = a$
if the denominator $D(a) = 0$ and the numerator $N(a)$ is a non-zero number.

Since $\csc x = \dfrac{1}{\sin x}$, $\cot x = \dfrac{\cos x}{\sin x}$, $\sec x = \dfrac{1}{\cos x}$ and $\tan x = \dfrac{\sin x}{\cos x}$,
$\csc x$ and $\cot x$ will have the same vertical asymptotes since they have the same denominator,
and $\sec x$ and $\tan x$ will have the same vertical asymptotes since they have the same denominator.

If you want to find the vertical asymptotes of $y = A\csc(Bx+C)+D$ or $y = A\sec(Bx+C)+D$,
there is an algebraic way and a graphical way.

Algebraic way

The algebraic way relies on your knowledge of which angles sine and cosine are equal to 0 at.

$y = A\csc(Bx+C)+D = \dfrac{A}{\sin(Bx+C)}+D$ has a vertical asymptote at all $x-$values where $\sin(Bx+C)=0$.
From section 4.2 and the unit circle, the sine of an angle is a $y-$coordinate on the unit circle,
and the $y-$coordinate is only 0 on the unit circle at the far left and far right of the unit circle.
These correspond to the angles which are coterminal with 0 or π.
However, all these angles are separated from each other by exactly one semi-circle or π radians.
So, the sine of an angle is 0 exactly when $x = 0 + k\pi = k\pi$ where $k \in \mathbf{Z}$.
So, $\sin(Bx+C)=0$ exactly when $Bx+C = k\pi$ where $k \in \mathbf{Z}$.
So, $y = A\csc(Bx+C)+D$ has vertical asymptotes where $Bx+C = k\pi$ where $k \in \mathbf{Z}$.
(Solve that last equation for x to get the equations of the vertical asymptotes.)

Since cosecant and cotangent have the same denominator (sine),
the vertical asymptotes of a cotangent graph can be found using the same equation.

Similarly, $y = A\sec(Bx+C)+D = \dfrac{A}{\cos(Bx+C)}+D$ has a vertical asymptote at all $x-$values where $\cos(Bx+C)=0$.
From section 4.2 and the unit circle, the cosine of an angle is an $x-$coordinate on the unit circle,
and the $x-$coordinate is only 0 on the unit circle at the top and bottom of the unit circle.
These correspond to the angles which are coterminal with $\dfrac{\pi}{2}$ or $\dfrac{3\pi}{2}$.
However, all these angles are separated from each other by exactly one semi-circle or π radians.
So, the cosine of an angle is 0 exactly when $x = \dfrac{\pi}{2} + k\pi$ where $k \in \mathbf{Z}$.

So, $\cos(Bx+C)=0$ exactly when $Bx+C = \dfrac{\pi}{2} + k\pi$ where $k \in \mathbf{Z}$.

So, $y = A\sec(Bx+C)+D$ has vertical asymptotes where $Bx+C = \dfrac{\pi}{2} + k\pi$ where $k \in \mathbf{Z}$.

(Solve that last equation for x to get the equations of the vertical asymptotes.)

Since secant and tangent have the same denominator (cosine),
the vertical asymptotes of a tangent graph can be found using the same equation.

Graphical way

The graphical way relies on your knowledge of the shape of sine and cosine graphs,
how the middle $y-$value, starting point/phase shift and quarter-period are related,
and how the sine and cosecant graphs are related, and how the cosine and secant graphs are related.

Remember that $y = A\csc(Bx+C)+D$ has a vertical asymptote
at all $x-$values where $y = A\sin(Bx+C)+D$ passes through its middle $y-$value.
That is, at the starting point/phase shift,
and then every 2^{nd} quarter-period either to the left or to the right of that.
So, the equations of the vertical asymptotes would be
$x = \text{PHASE SHIFT} + 2(\tfrac{1}{4}\text{PERIOD})k$ or $x = \text{PHASE SHIFT} + (\tfrac{1}{2}\text{PERIOD})k$ where $k \in \boldsymbol{Z}$.

Similarly, $y = A\sec(Bx+C)+D$ has a vertical asymptote
at all $x-$values where $y = A\cos(Bx+C)+D$ passes through its middle $y-$value.
That is, at the first quarter-period after the starting point/phase shift,
and then every 2^{nd} quarter-period either to the left or to the right of that.
So, the equations of the vertical asymptotes would be
$x = \text{PHASE SHIFT} + \tfrac{1}{4}\text{PERIOD} + 2(\tfrac{1}{4}\text{PERIOD})k$ or $x = \text{PHASE SHIFT} + \tfrac{1}{4}\text{PERIOD} + (\tfrac{1}{2}\text{PERIOD})k$ where $k \in \boldsymbol{Z}$.